

MAS125 Calculus II 2006-2007

Problem sheet for exercise class 8

- The instructor will present only one starred problem, leaving you more time to work on the other problems, which are similar to the ones in the assessed coursework.
- Since there is not time to do all the problems in the tutorial, it is important to study the solutions once they become available online at 3pm on Friday.
- Although tutorial problems are unassessed, they are in the same style as the problems in the final exam, so you are advised to study them carefully.

(*)Problem 1: If one substitutes polar coordinates $x = r \cos \theta$ and $y = r \sin \theta$ into the differentiable function $w = f(x, y)$, use the chain rule to show that

$$\frac{\partial w}{\partial r} = f_x \cos \theta + f_y \sin \theta, \quad \frac{1}{r} \frac{\partial w}{\partial \theta} = -f_x \sin \theta + f_y \cos \theta.$$

Solve these equations to express f_x and f_y in terms of $\partial w / \partial r$ and $\partial w / \partial \theta$ and deduce

$$(f_x)^2 + (f_y)^2 = \left(\frac{\partial w}{\partial r} \right)^2 + \frac{1}{r^2} \left(\frac{\partial w}{\partial \theta} \right)^2.$$

Problem 2: Express dw/dt as a function of t both by using the chain rule and by expressing w in terms of t directly for

$$w = \frac{x}{z} + \frac{y}{z}, \quad x = \cos^2 t, \quad y = \sin^2 t, \quad z = 1/t.$$

Problem 3: Use the chain rule to express $\partial w / \partial u$ and $\partial w / \partial v$ as functions of u and v for

$$w = xy + yz + xz, \quad x = u + v, \quad y = u - v, \quad z = uv.$$

Show that one gets the same result by expressing w in terms of u and v directly.

Problem 4: Find the gradient of the function $f(x, y)$ at the point P_0 indicated. In each case, sketch the gradient together with the level curve that passes through the point.

$$(a) \quad f(x, y) = y - x, \quad P_0(2, 1); \quad (b) \quad f(x, y) = \ln(x^2 + y^2), \quad P_0(1, 1).$$

Problem 5: Find the derivative of f at the point P_0 in the direction of the vector $\mathbf{A}$ indicated.

$$(a) \quad f(x, y) = 2xy - 3y^2, \quad P_0(5, 5), \quad \mathbf{A} = 4\mathbf{i} + 3\mathbf{j};$$
$$(b) \quad f(x, y, z) = xy + yz + xz, \quad P_0(1, -1, 2), \quad \mathbf{A} = 3\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}.$$

Problem 6: Find the directions in which the function $f(x, y) = x^2 + xy + y^2$ increases and decreases most rapidly at $(-1, 1)$ and find the derivatives in those directions.

Problem 7: Determine the equation of the tangent line at the point indicated for each of the curves below. In each case, sketch the curve and the tangent line.

$$(a) \quad xy = -4, \quad (2, -2); \quad (b) \quad x^2 - y = 1, \quad (\sqrt{2}, 2).$$

Problem 8: Find the equation of the tangent plane and normal at the point P_0 for the surfaces:

$$(a) \quad x^2 + y^2 + z^2 = 3, \quad P_0(1, 1, 1); \quad (b) \quad 2z - x^2 = 0, \quad P_0(2, 0, 2).$$